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Guidance for the Filtering of Dynamic Force, Pressure, Acceleration (and Other) Signals

Written By

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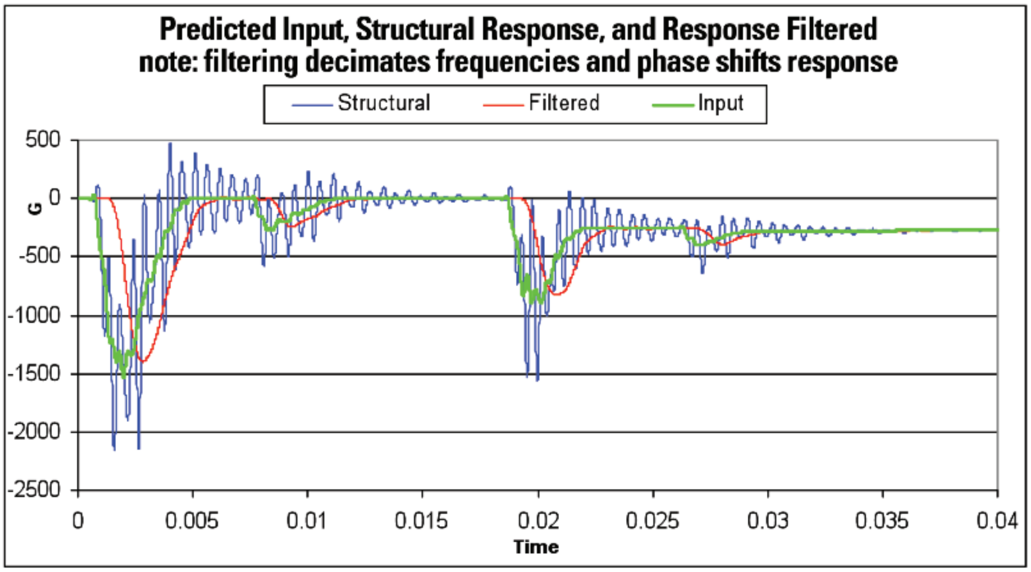
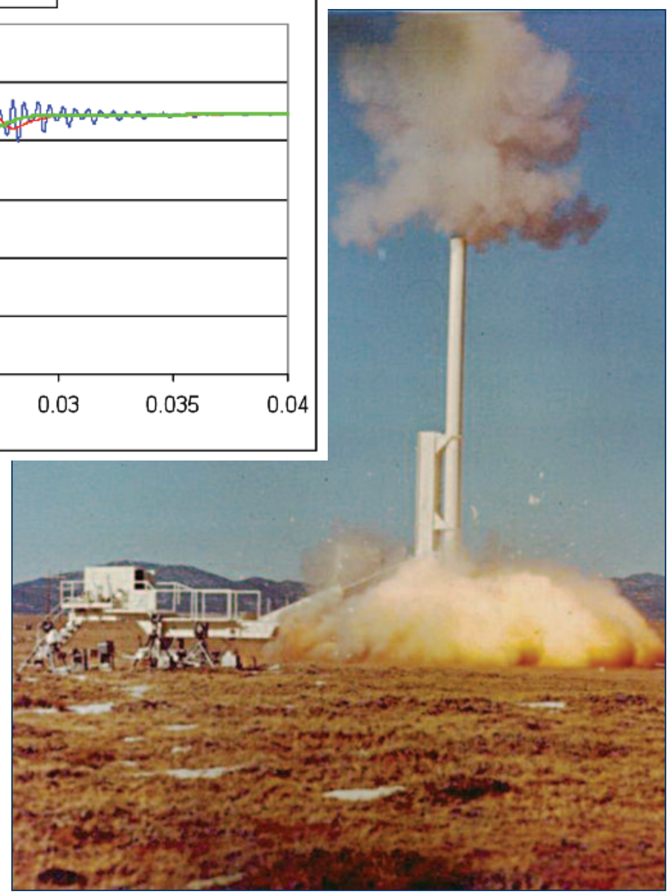




Filters are frequency selective devices described by their types (low-pass, high-pass, band-pass, and band-reject), their characterization (e.g., names such as Bessel, Butterworth, Chebyshev), and their complexity (the filter order, which controls their rate of signal attenuation with frequency). When using force, pressure, or acceleration transducers to measure the loading to or response of structural systems, linear, low-pass filters are typically employed to condition their signals. These filters can perform any or all of the following functions: (1) eliminate the transducer's own, internal, high-frequency structural resonances while preserving its undistorted, low-frequency signal region of interest; (2) eliminate the possibility of aliasing where, due to inadequate sampling rate, high frequency data "folds over" and corrupts this just mentioned low-frequency signal region; and (3) more effectively utilize the measurement system's data bandwidth and storage capacity. A brief introduction to analog filters and their associated terminology is first provided, and then this work focuses on the selection of appropriate analog, low-pass filters for any given user application. Aside from preserving the transducer's signal frequency content, guidance will be also provided towards preserving its wave shape. The principal contribution of this work to the literature will be a table that enables filters to simply and quickly be selected to support structural measurements. A limited discussion of digital filtering for data post-processing will culminate this effort.

Introduction:

If analysis of measured data is only required in the time domain, the signal simply has to be sampled fast enough to visualize the highest frequency of interest. Sampling a signal at 10 times this highest frequency will define the peak value of that frequency within 5%. This error criterion results from that fact that sampling 10 times per cycle for any frequency will miss its peak by no more than 18 degrees. The cosine of 18 degrees is 0.951.

When making dynamic measurements with force, pressure, or acceleration transducers, it is common to incorporate low-pass filters at some location in front of the digitizer in the measurement system. This filtering is implemented to: (1) eliminate the transducer's own, internal, high-frequency structural resonances while preserving its undistorted, low-frequency signal region of interest; (2) eliminate the possibility of aliasing [6] where, due to inadequate sampling rate, high frequency data "folds over" and corrupts this just mentioned low-frequency signal region; and (3) more effectively utilize the measurement system's data bandwidth and storage capacity. This last consideration is particularly important in environments such as transportation where 100s or 1000s of transducer-based data channels might be recorded during a specific test. All of these channels have to be recorded on, identified on, and accurately retrieved from data storage media.

A specific filtering technique, Sigma-Delta [15] filtering, while useful in some applications is only mentioned here in passing. This technique involves extreme oversampling, subsequent digital filtering, and then data decimation. For very large channel counts an already grievous data storage and retrieval issue can be aggravated. More important, the effective filtering is a combination of the manufacturers front end analog filter (often first-order, low-pass) and the subsequent digital filter. This combination is not unique. We will then consider the more classical filters as will be discussed in subsequent paragraphs.

The amplitude response of an ideal, low-pass filter [1] would uniformly pass all frequencies to some upper limit and then completely eliminate frequencies above that limit. The phase response of this ideal, low-pass filter would be perfectly linear to this same upper frequency bound. Such a filter would maintain signal frequency fidelity to this upper limit while introducing only a time delay in the output signal. Wave shape of the signal over this frequency range would be preserved. The resultant time delay could be calculated from the slope of the phase (radians) versus frequency (radians/second) response. This slope is often referred to as the Group Delay or Time Delay [10] of the filter. A linear phase response results in a constant Group Delay.

Ideal filters do not exist. Figures 1a and 1b are the amplitude and phase responses for both a 6-pole Chebyshev filter and a 6-pole Bessel filter with the same -3dB frequency. The Chebyshev best approximates the amplitude response of the ideal filter while the Bessel best approximates the ideal phase response. Thus, data filtering always involves some compromise.

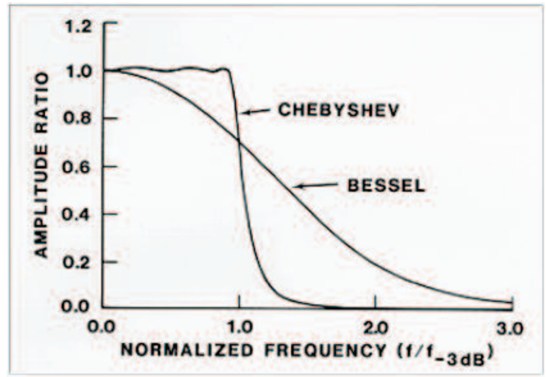


FIGURE 1A. COMPARATIVE AMPLITUDE RESPONSE

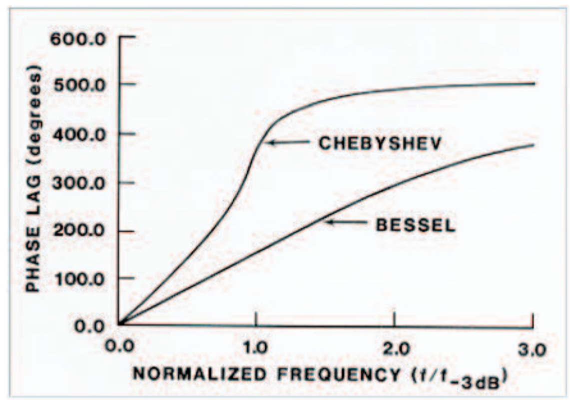


FIGURE 1B. COMPARATIVE PHASE RESPONSE

FIGURE 1. RESPONSE OF TWO DIFFERENT 6-POLE FILTER TYPES

The terminology that describes filters is often confusing. Figure 2 is a simple RLC circuit. The output measured across the capacitor is in fact a low-pass filter.

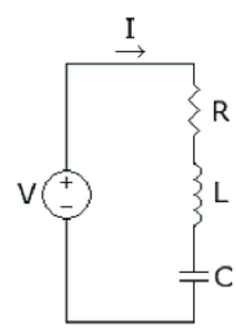


FIGURE 2. RLC CIRCUIT

The transfer function across this capacitor is [2]:

$$H(s) = (Vo/Vin) = 1/(LCs^2 + RCs + 1) \quad (1)$$

The degree of the denominator of H(s) in this equation is n = 2; therefore, this is a second order filter. Solving for the roots of the denominator provides values for the poles of the filter. Each pole will provide a slope of -1 on a log amplitude versus log frequency plot, which is equivalent to -6 dB/octave or -20 dB/decade response attenuation [4]. The above H(s), being 2nd order, would then produce an ultimate slope of -2 on a log amplitude versus log frequency plot, equivalent to -12dB/octave or -40 dB/decade attenuation.

If H(s) is realizable, we are able to substitute $s = j\omega$ [3] resulting in a complex function of frequency $H(j\omega)$. We typically plot this frequency response function in the form of Bode plots, i.e., amplitude and phase versus frequency plots. Figure 3 below illustrates the output across the capacitor (C) of Figure 2, verifying it to be a low-pass filter. The exact shape of the amplitude (top) and phase (lower) responses depends on the damping parameter ζ and the natural frequency ω_n. While not the focus of this work, the output across the resistor (R) in Fig. 2 would be band-pass in form and the output across the inductor (L) would be high-pass.

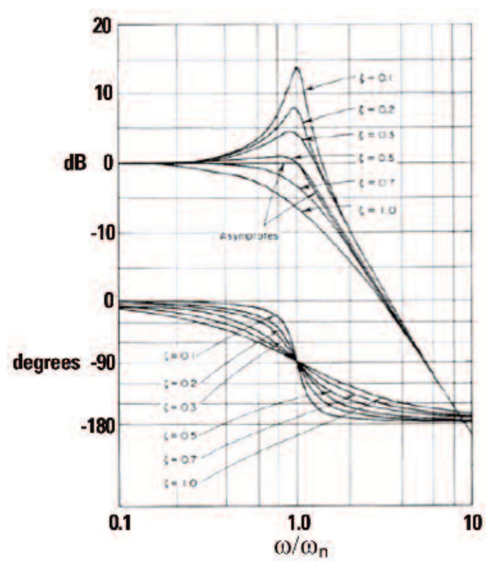


FIGURE 3. BODE PLOTS ACROSS CAPACITOR IN FIGURE 2

The bandwidth of a filter is typically specified in terms of its -3dB frequency. For a simple, 1-pole, low-pass (RC) filter, this -3 dB frequency, in radians/second, is 1/(RC). That is, at the -3dB frequency only 0.707 of the input signal is passed. For all filter configurations other than RC, the -3 dB frequency has little physical significance. However, by convention we specify filter bandwidth in terms of this -3dB frequency.

Low and high-pass filters are primarily implemented with operational amplifiers. A generic, second order, active filter configuration [5] is shown in Fig. 4. For example, a low-pass filter can be configured by making G2/G4 capacitors and G1/G3 resistors. A 4-pole filter would require two such stages in series, a 6-pole three stages, etc. The values of G1, G2, G3, and G4 for each stage would depend on the character of the specific filter desired. Each stage provides 180 degrees of phase shift; e.g. a 3-stage or 6-pole filter would have an associated 3 x 180 or 540 degrees of total phase shift.

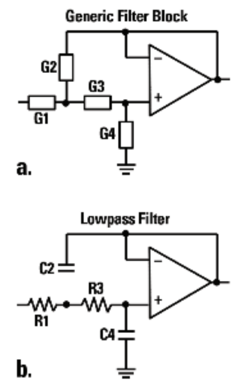


FIGURE 4. 2-POLE GENERIC LOW-PASS ACTIVE FILTER

Filter Characterizations:

There are four classic analog filter characterizations: Butterworth, Chebyshev, Elliptic and Bessel. The Elliptic will be mentioned but for reasons presented below the other three (3) will be characterized in great detail.

Butterworth [11]: The first and probably best-known filter approximation is the Butterworth or maximally-flat amplitude response. It exhibits a nearly flat pass band with no ripple. Its roll-off is smooth and monotonic. It has a reasonably linear phase response.

Chebyshev [12]: The Chebyshev response follows a mathematical strategy for achieving a faster roll-off by allowing ripple in the amplitude response. As the ripple increases (bad), the roll-off becomes sharper (good). The Chebyshev response is an optimal trade-off between these two parameters. Chebyshev filters, where the ripple is only allowed in the pass band, are called type 1 filters. Chebyshev filters that have ripple only in the stop band are called type 2 filters, but they are seldom used. Chebyshev filters have a relatively nonlinear phase response.

Bessel [13]: The Bessel filter has nearly perfect phase linearity in the pass band. However, its amplitude roll-off is slower than either the Butterworth or Chebyshev filter for an equivalent order (number of poles).

Elliptic [14] often called Cauer: The cut-off slope of an elliptic filter is steeper than that of a Butterworth, Chebyshev, or Bessel, but the amplitude response has ripple in both the pass band and the stop band. In addition, its phase response is highly nonlinear. If the primary concern is to pass frequencies falling below a certain frequency limit and reject frequencies above that limit, regardless of phase shifts or ringing, the elliptic response will optimally perform that function. However, we will ignore this filter because its highly nonlinear phase greatly distorts complex time signals.

Selection Criteria:

The establishment of low-pass filter selection criteria has to consider two requirements. One requirement dictates that at a preselected upper frequency limit the filter must attenuate the signal by some specified amount. The second requirement dictates that over some range of frequencies below this limit the filter must maintain both flat amplitude and linear phase response to preserve signal wave shape.

Specific requirements must be generated in order to establish these criteria. To initiate this discussion, these requirements will be specified as: (1) 95 percent (26dB) signal attenuation by some selected upper frequency limit and (2) both flat amplitude within 5 percent and linear phase within 5 degrees over some specified range of lower frequencies. Although the final outcome of this study will allow us to apply flexibility to these criteria, their justification as a starting point will subsequently be provided.

Right (Fig. 5) are three color coded plots for four configurations of 4-pole, low-pass filters. All of these filters are normalized to provide 95% amplitude attenuation (see top plot) at a frequency value of 1.0. The three groupings of plots respectively are amplitude versus frequency, phase (0 to 360 degrees) versus frequency, and time or group delay versus frequency. Within a specific grouping, from top to bottom, is a 0.5 dB type 1 Chebyshev (green), 0.1 dB type 1 Chebyshev (dashed magenta), Butterworth (red), and Bessel (dotted blue) characterization. For all plots in this figure both the vertical axes (ordinates) and horizontal axes (abscissas) are shown on a linear scale.

Looking at the plots of Fig. 5, several facts are apparent. First, for the 4-pole filters, Fig. 5 (top) shows that the amplitude response of the Chebyshev filters are much more frequency selective than the others. As predicted, the Butterworth is intermediate in selectivity and the Bessel is the least selective. Figure 5 (center) does not clearly delineate phase linearity or lack thereof. However, the more constant group delay in the bottom plot of Fig. 5 for the Bessel filter shows it to have the most linear phase, the Butterworth intermediate, and the Chebyshev filters to have the most nonlinear phase characteristics.

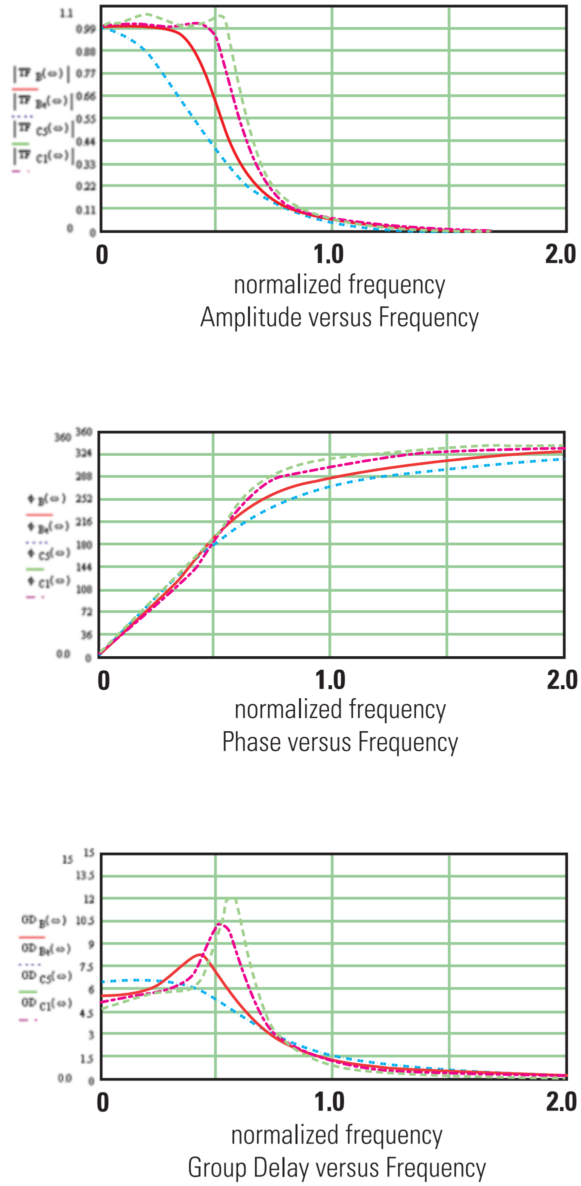
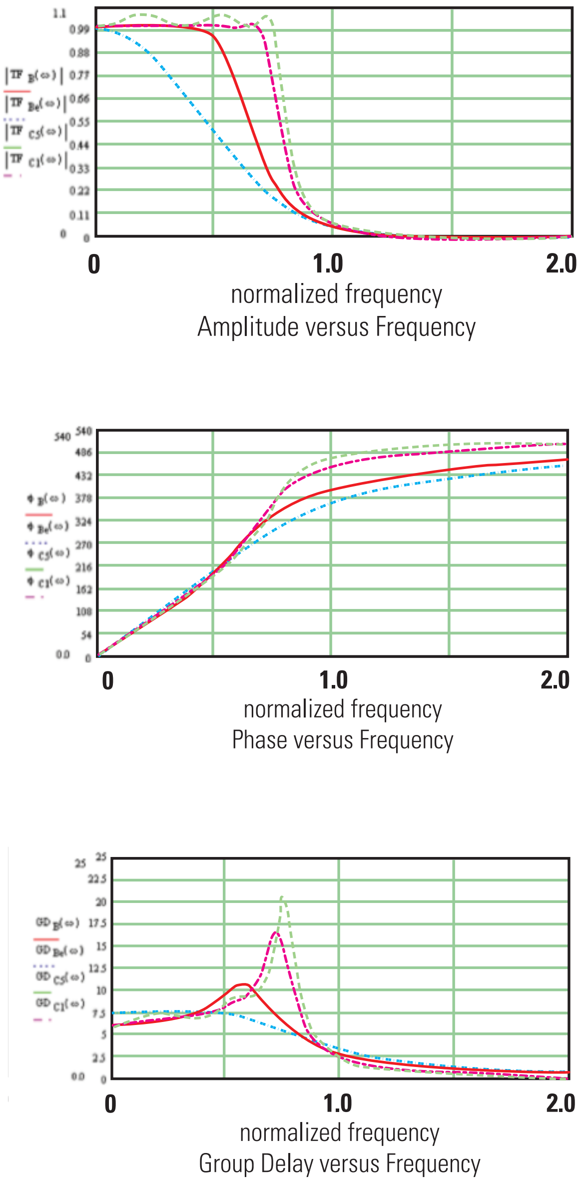


Figure 5. Amplitude, Phase, and Group Delay for Various 4-Pole, Low-Pass Filter Configurations



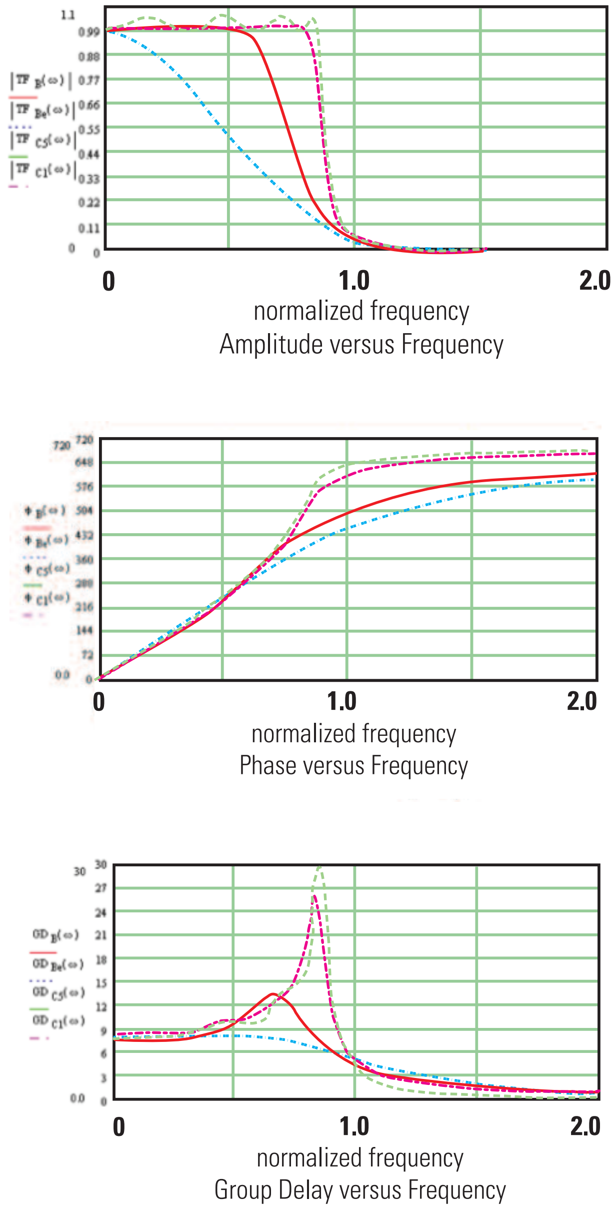


FIGURE 6. AMPLITUDE, PHASE, AND GROUP DELAY FOR VARIOUS 6-POLE, LOW-PASS FILTER CONFIGURATIONS

FIGURE 7. AMPLITUDE, PHASE, AND GROUP DELAY FOR VARIOUS 8-POLE, LOW-PASS FILTER CONFIGURATIONS

Figures 6 and 7 repeat this same sequence of plots for 6-pole and 8-poles versions of these same filters. The same trends as in Fig. 5 are observed to occur.

Having presented these global observations, what is required is to provide the test engineer or technician with a quick and simple tool to select an appropriate filter for his/her application. The tables in the appendix parameterize these plots to facilitate this selection process. The tables will next be explained along with application examples and a methodology to implement them and even increase their flexibility.

Tables Explanation and Implementation

Among other things, the successful design of an instrumentation system has to include: (1) the sampling rate of the existing digitizer or that rate which will ultimately be required, (2) the data bandwidth required or that which will result as a byproduct of filter selection, and (3) the existing available filter characteristic and order or that which will ultimately be required for compatibility with the data sampling rate. Considering only item (3), specific filters characteristics tabulated in the appendix include Chebyshev type 1 with 0.5 dB ripple in its pass band, Butterworth, and Bessel. For completeness, data are tabulated for 2, 4, 6, and 8 poles. However, pragmatically only 4, 6, and 8 poles attenuate or roll-off fast enough to be considered as practical anti-aliasing filters.

To enter the tables we either have to start with an initial requirement on amplitude "flatness" or constancy and/or a maximum acceptable phase nonlinearity. The first sheet (no.1) allows us to enter with an amplitude flatness requirement and the second (no.2) with a maximum phase nonlinearity requirement. The following examples will illustrate their use.

Example 1: Assume that we need to preserve the frequency content of a signal to 2000 Hz while also minimizing its wave shape distortion over this same bandwidth. We decide that maintaining frequency response flat within 5% and phase response linear (based on the filter's initial phase-frequency slope) within 5 degrees over this frequency range will satisfy both these requirements. We have 4-pole (i.e., 4th order) 0.5 dB Chebyshev, Butterworth, and Bessel filters available to select from. The goal is to determine which of these filter characteristics best satisfies our requirements while minimizing our sampling rate, thereby also minimizing the quantity of data we have to store.

Solution: Looking at sheet no.1, we enter at the top with the column "flat to 5%" and observe when satisfying the required criteria with 4th order filters the Butterworth can get us to within 35.82% of its 95% attenuation frequency, the Bessel to within 12.91%, and the Chebyshev to within 16.07%. Now we have to determine how well phase linearity is maintained over these same frequency ranges. We go to the bottom part of this same sheet in the 5% column and find that the corresponding phase nonlinearity is 13.31 degrees for the Butterworth, -0.000 degrees for the Bessel, and 3.04 degrees for the Chebyshev. Thus, both the Bessel and the Chebyshev have satisfied our criteria, but the Butterworth has not yet accomplished this. Since the Butterworth is limited in this application by its phase nonlinearity, we enter sheet no. 2 at the top with "5 degrees" max phase nonlinearity. We see that the 4-pole Butterworth can get us to 27.16% of the 95% attenuation frequency while satisfying this 5 degree requirement. We go to the bottom part of this sheet under the 5 degrees column and confirm the Butterworth is flat over this region within exactly 0.59%. Thus both criteria are satisfied. The conclusion is the Bessel satisfies our starting criteria to 12.91% of its 95% attenuation frequency, the Chebyshev to 16.07% and the Butterworth to 27.16%. Thus, given our selection choices, the Butterworth is the most optimum and can perform within our 5% amplitude requirement and our maximum 5% phase nonlinearity criteria while providing 95% amplitude attenuation at (2000Hz/.2716) or 7364 Hz. The minimum sampling rate required would be 14728 samples/second based on the Nyquist sampling criterion. The sampling rate required for the Bessel would be higher (2000/.1291) x 2 or 30984 samples/second as would the Chebyshev (2000/.1607) x 2 or 24891 samples/second based on this same criterion. *While this example doesn't prove it conclusively, it can be shown that when the goal is to both optimize amplitude "flatness" and phase linearity while minimizing sampling rate the* Butterworth *will always be the better performing filter among the three being evaluated.* <u>*It should also be noted that neither 5% deviation from amplitude flatness or 5 degrees deviation from phase linearity imply large errors in the time history. These errors only occur only at the maximum frequency in the signal spectrum of interest. Signals other than a sine wave are typically complex and contain multiple frequencies. Thus, the composite signal error, while dependent on the specific wave shape, would typically be very small.*</u>

Example 2: We will again work the same problem as Example 1 above but this time require amplitude response flat within 2% and phase linearity (based on initial phase-frequency slope) within 2 degrees to 2000 Hz from our 4-pole Butterworth filter.

Solution: Looking at sheet no.1, we enter at the top with the column "flat to 2%" and observe while satisfying this criterion the Butterworth can get us to within 31.75% of its 95% attenuation frequency. However, the lower portion of this sheet in the 2% column shows that our phase will be 8.68 degrees from linearity (> the 2 degrees allowed). We therefore enter sheet no. 2 at the top with "2 degrees" phase linearity. We see that the 4-pole Butterworth can get us to 20.74% of the 95% attenuation frequency at the 2 degree phase nonlinearity point. We then go to the bottom part of this sheet under the 2 degrees column to confirm the Butterworth is flat over this region to exactly 0.068%. Thus, this Butterworth can perform within our 2% amplitude requirement and our 2% maximum phase nonlinearity criteria and provide 95% amplitude attenuation at (2000Hz/.2074) or 9643 Hz. The minimum sampling rate required would be 19286 samples/second based on the Nyquist criterion. Obviously this rate would be rounded to a more even number (e.g., 20,000 samples/second). We will expand this example further by referring to sheet no. 3. Sheet no. 3 shows that the -3dB point for this filter would be 9643 x .473 or 4561 Hz. Thus a 4-pole Butterworth with a -3dB frequency at 4561 Hz would satisfy the amplitude and phase requirements of this example and provide 95% attenuation at 9643 Hz. Again, a more convenient – 3dB value could be selected by applying whatever value of conservatism the customer desired.

Example 3: We have a digitizer with a sampling rate of 250,000 samples per second. Signal frequency content is to be optimized and wave shape reproduction is important. What filter type and -3dB point should we specify to best satisfy this requirement?

Solution: By now we recognize that among the filters characterized the Butterworth has the optimum performance when considering both its amplitude and phase characteristics. It can also be inferred that higher orders filters offer better overall performance than lower order filters. We will go directly to the 8th order Butterworth with requirements (justified earlier) for frequency response flat within 5% and maximum phase nonlinearity (based on initial phase-frequency slope) of 5 degrees over our frequency range, which has yet to be determined. This problem can be worked both ways but the Butterworth filter characteristic will become limited by nonlinear phase before it deviates from flat amplitude response. Therefore we will go directly to sheet no. 2 with "5 degrees phase linearity" and see that we can operate to 34.8% of the 95% attenuation frequency while satisfying this requirement. We next go to the bottom part of this sheet under the 5 degrees column to confirm the Butterworth is flat over this region to exactly 0.001%. If we specify 95% (26 dB) attenuation at the Nyquist frequency (250,000/2) or 125,000 Hz, we can optimize our filter selection with an 8th order (8-pole) Butterworth and satisfy our requirements to 125,000 x .348 or 43,500 Hz. From sheet no. 3 the -3dB filter value would be 125,000 x 0.687 or 85,875 Hz. Again, a close but more convenient value could be selected. *By contrast, if we had considered an 8th order Bessel, sheet no. 1 shows it would become limited by its 5% amplitude deviation at 14.74% of the 95% attenuation frequency. We could check the phase in sheet no.1, but Bessel filters are always limited by their amplitude deviation from flatness. Thus, for the same sampling rate the Bessel filter in this instance affords only (.147/.348) or 42% of the bandwidth provided by the Butterworth.*

Why is 95% Attenuation at the Nyquist Frequency Generally Enough?

First, there is no hard and fast rule on how much attenuation is required at the Nyquist frequency. An absolute answer could only be based on apriori knowledge of the signal, a specific filter characteristic and order, and the allowable aliasing error contribution. Let's look at Example 3 as a basis for discussion since an 8th order Butterworth filter has a relative sharp roll-off or attenuation curve. Let's round off the -3dB frequency of the filter in this example to 86,000 Hz. Note that 86,000 subtracted from the Nyquist frequency (125,000 – 86,000) is 39,000 Hz. Let's then assume that we have data signal frequency content at 125,000 + 39,000 or 164,000 Hz. This spectral content would be folded back or aliased to 86,000 Hz (the filters -3dB frequency). A simple calculation shows that at 164,000/86,000 or 1.91 times the -3dB point of this filter less than 0.6% of this 164,000 Hz signal content is folded back or aliased to the filter's -3dB point. This aliased frequency content would be added to the spectral content at 86,000 Hz which is already in error by virtue of being attenuated by 29.3% (-3dB). In this example the error contribution due to aliasing would be inconsequential relative to the 29.3% existing error due to filter signal attenuation. This is typically the case.

Pyroshock:

Pyroshock is a unique type of high frequency mechanical shock mentioned here only because a currently proposed revision to a specific military standard 810G [9] requires approximately 50 dB attenuation from the filter at the Nyquist frequency of the sampled data. Whether this value is overly conservative or not can be debated, but 50dB corresponds to an attenuation of 1 part in 316 (1/316th). The tables in the appendix can still be used effectively. For an 8th order Butterworth this amount of attenuation occurs at 2.05 times the -3dB frequency or 1.41 times the frequency where 95% attenuation occurs in this filter. Thus, if you use the developed tables for the 8th order Butterworth, perform the standard calculations based on 95% or 26dB attenuation at the Nyquist frequency, and increase the resultant sampling rate by 45% [((2.05 – 1.41) / 1.41) x 100], military standard 810G will be satisfied. For a 4th order Butterworth an analogous calculation would show the sampling rate needs to be increased by 100%.

Subsequent Digital Filtering:

First, it should be clearly stated that if analog data become aliased during the measurement process, no amount of subsequent digital filtering can correct this situation. Digital filtering is only used to further limit the data bandwidth after valid data are initially recorded. Only a brief discussion of digital filtering is provided here. This discussion is further limited to Butterworth characteristic digital filters since these are routinely available in commercial software packages such as MATLAB® and LabView®. Other digital filter types are available.

The previous work has thus far enabled the recording of a signal with a given data bandwidth that can be certified to be within known bounds of amplitude flatness and phase linearity. However, for any complex signal, signal content still exists above this bandwidth. For example, signal content exists at the -3dB point of the measurement system, which is 29.3% reduced from its true value. Higher frequencies result in further attenuation and increased phase nonlinearities. If it is desired to further constrain signal content to a specific upper frequency bound, subsequent digital filtering can help.

Zero phase digital filters can be achieved. The analogous resultant analog filter characteristics would be equivalent to:

$$[H(j\omega)H^*(j\omega)] = |H(j\omega)|^2 \qquad (2)$$

where the * denotes a complex conjugate. An equivalent digital filter would have no phase shift and an amplitude response which is the square of that of the analog filter. For example, an 8-pole digital Butterworth filter, when requested with a zero phase option, would produce a 16-pole roll off. This is equivalent to 96dB/octave or 320dB/decade attenuation. Thus it is possible to closely limit any recorded data bandwidth to a selected upper bound by eliminating the higher frequencies in the signal where distortion is occurring. Digitally this is achieved by passing the sampled data through the digital Butterworth filter [7,8], passing the reversed output data through the filter a second time, and reversing the order of this output a final time.

There are other unique features of this filter, but it is easiest to assess its effectiveness in application. Assume that data are recorded, as in example #2, with amplitude response flat within 2% and phase linearity (based on initial phase-frequency slope) within 2 degrees to 2000 Hz using our 4-pole Butterworth filter. Next, assume it is desired to limit data frequency content as much as possible to 2000 Hz to enable model correlation. I can first perform a Fourier transform on this data and note its spectral content. I can now pass this digitized record through a high order, zero phase, digital Butterworth filter and progressively iterate its -3dB frequency lower in value. When spectral content at 2000 Hz just begins to become influenced, the iterations are stopped. The extreme attenuation achievable in the zero phase digital filter will eliminate the majority of the signal content above 2000 Hz without introducing amplitude or phase distortion over the data bandwidth of interest

<u>Conclusion:</u>

While analog filters are often integrated into measurement systems, their selection process lacks precise guidelines. The work presented here has:

1. explained the technical basis of analog filters,
2. proposed a set of guidelines for filter implementation,
3. provided justification for these guidelines,
4. provided a set of parameterized tables to greatly simplify guideline implementation,
5. enabled enough flexibility in these tables to allow a designer to modify these guidelines,
6. provided typical application examples, and
7. described the complimentary role that digital filtering can play once meaningful data are acquired.

It is hoped this effort will greatly simplify the filter selection and/or design process for the measurement engineer.

Acknowledgement: These tables were generated under my direction a number of years ago by Mr. Manoj Gopalan, a former TCU student. His efforts are appreciated. I have made a number of checks on their accuracy and they have been displayed and validated for efficacy in numerous TCU Continuing Education programs.

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APPENDIX
FILTER SELECTION TABLES

SHEET 1
Filters Sorted By Flat Amplitude

All Filters Scaled to have 95% Attenuation at $\omega = 1$

Filter Type	Order	Flat to within			
		10%	**5%**	**2%**	**1%**
Butterworth	2nd	15.57	12.83	10.08	8.45
Butterworth	4th	39.46	35.82	31.75	29.06
Butterworth	6th	53.8	50.43	46.54	43.87
Butterworth	8th	62.82	59.85	56.35	53.91
Bessel	2nd	10.03	7.11	4.51	3.2
Bessel	4th	18.39	12.91	8.13	5.74
Bessel	6th	20.67	14.47	9.09	6.42
Bessel	8th	21.08	14.74	9.26	6.53
0.5dB Chebyshev	2nd	21.23	10.03	5.67	3.93
0.5dB Chebyshev	4th	57.67	16.07	8.81	6.06
0.5dB Chebyshev	6th	76.24	14.62	7.97	5.48
0.5dB Chebyshev	8th	85.29	12.4	6.75	4.63

Filter Type	Order	Corresponding Phase Non Linearity			
		10%	**5%**	**2%**	**1%**
Butterworth	2nd	5.957	3.921	2.134	1.317
Butterworth	4th	18.475	13.311	8.687	6.34
Butterworth	6th	31.875	24.615	17.821	14.139
Butterworth	8th	46.019	37.152	28.601	23.778
Bessel	2nd	-0.2917	-0.056	-0.0060414	-0.00109414
Bessel	4th	-0.0025	-0.000114	-0.00000195	0
Bessel	6th	-0.0000067	0	0	0
Bessel	8th	0	0	0	0
0.5dB Chebyshev	2nd	22.1705	3.3499	0.6133	0.204
0.5dB Chebyshev	4th	61.9076	3.038	0.557	0.185
0.5dB Chebyshev	6th	111.551	2.854	0.527	0.176
0.5dB Chebyshev	8th	166.682	2.761	0.512	0.171

SHEET 2
Filters Sorted By Linear Phase
All Filters Scaled to have 95% Attenuation at $\omega = 1$

Filter Typez	Order	Phase Linear to within			
		10°	5°	2°	1°
Butterworth	2nd	38.24	14.29	9.84	7.66
Butterworth	4th	33.03	27.16	20.74	16.75
Butterworth	6th	40.02	32.86	24.82	19.9
Butterworth	8th	42.77	34.8	26.11	20.88
Bessel	2nd	23.17	19.23	15.38	13.12
Bessel	4th	54.79	48.61	42.24	38.3
Bessel	6th	74.31	67.82	60.97	56.65
Bessel	8th	87.25	80.92	74.16	69.84
0.5dB Chebyshev	2nd	14.78	11.51	8.43	6.68
0.5dB Chebyshev	4th	26.82	19.51	13.78	10.78
0.5dB Chebyshev	6th	26.31	18.35	12.78	9.96
0.5dB Chebyshev	8th	23.48	15.86	10.97	8.53

Filter Type	Order	Corresponding Amplitude Attenuation			
		10°	5°	2°	1°
Butterworth	2nd	67.605	7.4	1.819	0.678
Butterworth	4th	2.712	0.586	0.068	0.012
Butterworth	6th	0.335	0.032	0.001105	0.0000889
Butterworth	8th	0.025	0.000837	0	0
Bessel	2nd	43.554	33.178	22.681	16.897
Bessel	4th	66.994	57.554	46.543	39.519
Bessel	6th	80.886	73.881	64.924	58.679
Bessel	8th	89.016	84.232	77.672	72.791
0.5dB Chebyshev	2nd	-5.166	-5.708	-3.936	-2.681
0.5dB Chebyshev	4th	-4.692	-5.821	-4.131	-2.834
0.5dB Chebyshev	6th	-4.005	-5.881	-4.255	-2.929
0.5dB Chebyshev	8th	-3.427	-5.904	-4.322	-2.979

SHEET 3
Relationship between 3dB points and 95% attenuation points

Filter Type	Order	Ratio of 3dB Point to 95% attenuation point
Butterworth	2nd	0.22375
Butterworth	4th	0.47302
Butterworth	6th	0.60709
Butterworth	8th	0.68776
Bessel	2nd	0.17781
Bessel	4th	0.32317
Bessel	6th	0.36861
Bessel	8th	0.37784
0.5dB Chebyshev	2nd	0.26073
0.5dB Chebyshev	4th	0.61124
0.5dB Chebyshev	6th	0.78286
0.5dB Chebyshev	8th	0.86580

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